Wigner Function Formalism for Zero Magnetic Field Spin Dependent Resonant Tunneling Structures

Mehmet Burcin Unlu, Hong-Liang Cui

Department of Physics and Engineering Physics,

Stevens Institute of Technology, Hoboken, NJ 07030, USA

Abstract

We develop a Wigner function representation of the quantum transport theory of the conduction band electrons in Rashba effect resonant tunneling structures with a phonon bath. In narrow band gap heterostructures, spin splitting occurs mainly as a result of inversion asymmetry in the spatial dependence of the potential or as a result of external electric field. This "zero magnetic field spin splitting" is due to the Rashba term in the effective mass Hamiltonian. The quantum transport equations are derived using multi-band non-equilibrium Green's function formulation in generalized Kadanoff-Baym ansatz.

Zero magnetic field spin splitting [1] of the conduction band states in bulk and compound semiconductors has attracted great interest recently. The possibility of a spin polarized current source based on conventional nonmagnetic semiconductors has led many researchers to investigate the possibility of obtaining efficient spin-polarized current source, a necessity in spintronics. Bulk inversion asymmetry (as in zincblende semiconductors) dominates in large band gap semiconductors whereas the structural inversion asymmetry (the Rashba effect) becomes important in narrow band gap semiconductors.

The Rashba effect predicts a spin splitting of the conduction band linear in $k_{\|}$. For example, the spin splitting in the lowest conduction subband exceeds 0.02 eV for in-plane wave vector $k_{\|} = 0.05$ $\mathring{A}^{-1}$ [2]. A similar splitting occurs in the light hole band proportional to $k_{\|}$ and in the heavy hole band proportional to $k_{\|}^3$ [3]. The case of conduction band electrons is more attractive for the spin transport devices since these have a much longer spin relaxation time.

Wigner function modeling of charge transport in multi-barrier resonant tunneling structures has been quite popular in the literature due to its success in dealing with the dissipation and the open boundary conditions [4]. Similarly, it is expected that one should be able to model the spin transport in the tunneling structures using the Wigner function.

We derive two conduction spin subband Wigner function equations for zero magnetic field resonant tunneling structures. These equations are derived for the first time in the literature. When the spin-orbit interaction is included in the single conduction band effective mass Hamiltonian, the Wigner function becomes a 2×2 matrix in spin subspace. The diagonal elements $f_{\uparrow\uparrow}$ and $f_{\downarrow\downarrow}$ are the spin up, spin down electron distribution functions respectively. The off-diagonal elements $f_{\uparrow\downarrow}$ and $f_{\downarrow\uparrow}$ describe the spin coherence. Note that $f_{\downarrow\uparrow} = f_{\uparrow\downarrow}^*$.

The Rashba effect resonant tunneling structures have been discussed from both theoretical

and experimental points of view in the literature. It was shown that an asymmetric double barrier resonant tunneling diode (DB RTD) might provide spin polarization above 50 % [5]. Koga et al proposed a triple barrier resonant tunneling diode (TB RTD) that can reveal a high degree of polarization 99.9 % [6]. Recently Ting and Cartoixa proposed the resonant inter-band tunneling spin filter [7].

If the z- axis is chosen as the growth direction, the effective mass Hamiltonian with the spin-orbit term can be written as

$$H = \hat{p}_z \frac{1}{2m^*(z)} \hat{p}_z + \frac{\mathbf{p}_\parallel^2}{2m^*(z)} + \frac{\alpha}{\hbar}(\sigma \times \mathbf{p})_z$$

where σ denotes the Pauli spin matrices, $\mathbf{p}$ is the momentum operator, $\mathbf{p}_\parallel = (p_x, p_y, 0)$ is the in-plane momentum vector. We assume that the in-plane momentum, $\mathbf{p}_\parallel$, is conserved across the device. α is the spin-orbit coupling constant (Rashba constant) and it is material dependent (inversely proportional to the energy gap and the effective mass) and proportional to the interface electric field along the growth direction. Therefore it is possible to tune this parameter by applying an external bias. The experimental value of the Rashba constant is in the order of $10^{-12} eV.m.$

The Hamiltonian for Rashba effect resonant tunneling structure can be written as,

$$H = \begin{bmatrix} H_{\uparrow\uparrow} & H_{\uparrow\downarrow} \\ H_{\downarrow\uparrow} & H_{\downarrow\downarrow} \end{bmatrix} = \begin{bmatrix} \hat{p}_z \frac{1}{2m^*(z)} \hat{p}_z + \frac{p_\parallel^2}{2m^*(z)} + E_c(z) & i\frac{\alpha}{\hbar}p_- \\ -i\frac{\alpha}{\hbar}p_+ & \hat{p}_z \frac{1}{2m^*(z)} \hat{p}_z + \frac{p_\parallel^2}{2m^*(z)} + E_c(z) \end{bmatrix} \qquad (1)$$

where $\hat{p}_z = -i\hbar\frac{\partial}{\partial z}$, $p_\pm = p_x \pm ip_y$, and $E_c(z) = E_c + V(z)$. E_c is the conduction band edge and $V(z)$ is the self-consistent potential.

Throughout the paper, σ denotes the spin index and there is a summation over repeated indices. The four-dimensional, (3+1), crystal momentum and its conjugate variable lattice coordinate are represented as $p = (\mathbf{p}, E)$, $r = (\mathbf{R}, T)$. Under the assumption that the self-energies are slowly varying with respect to the center of mass coordinates, generalized Kadanoff-Baym equation for in phase-space-energy-time domain can be written as [8],

$$
\begin{aligned}
i\hbar\frac{\partial}{\partial T}G^{<}_{\sigma\sigma'}(p,r) &= \frac{1}{(h^4)^2}\int dp_2 dr_2 K^c_{H_{\sigma\beta}}(p, r-r_2; r, p-p_2)G^{<}_{\beta\sigma'}(p_2, r_2) \\
&\quad +\Sigma^{>}_{\sigma\beta}(p,r)G^{<}_{\beta\sigma'}(p,r) - \Sigma^{<}_{\sigma\beta}(p,r)G^{>}_{\beta\sigma'}(p,r) \qquad (2)
\end{aligned}
$$

where

$$
K^c_A(p, r-r_2; r, p-p_2) = K^+_A(p, r-r_2; r, p-p_2) - K^-_A(p, r-r_2; r, p-p_2)
$$

and

$$
K^{\pm}_A(p, r-r_2; r, p-p_2) = \int dp_1 dr_1 \exp(\frac{i}{\hbar}p_1.(r-r_2))\exp(\frac{i}{\hbar}r_1.(p-p_2))A(p \pm \frac{p_1}{2}, r \mp \frac{r_1}{2}).
$$

The self-energy function for the electron-phonon scattering in phase-space-energy-time representation can be written as

$$
\Sigma^{>,<}_{\sigma\sigma'}(p,r) = \frac{i}{h^4}\int dq G^{>,<}_{\sigma\sigma'}(p+q)D^{>,<}(q). \qquad (3)
$$

Assuming the phonon bath is in equilibrium, the Fourier transforms of the phonon Green's functions can be written as,

$$D^{<}(\mathbf{q},E^{'})=-ihM_{\mathbf{q}}^{2}[(N_{\mathbf{q}}+1)\delta(E^{'}-\Omega_{\mathbf{q}})+N_{\mathbf{q}}\delta(E^{'}+\Omega_{\mathbf{q}})], \tag{4}$$

$$D^{>}(\mathbf{q},E^{'})=-ihM_{\mathbf{q}}^{2}[(N_{\mathbf{q}}+1)\delta(E^{'}+\Omega_{\mathbf{q}})+N_{\mathbf{q}}\delta(E^{'}-\Omega_{\mathbf{q}})] \tag{5}$$

where $M_{\mathbf{q}}$ is the electron-phonon scattering matrix element. Therefore, inclusion of the phonon scattering gives the following scattering functions

$$\Sigma_{\sigma\sigma'}^{<}=\sum_{\eta=+1,-1}\frac{1}{h^3}\int d\mathbf{q}G_{\sigma\sigma'}^{<}(\mathbf{p}+\mathbf{q},E+\eta\Omega_{\mathbf{q}},r)M_{\mathbf{q}}^{2}(N_{\mathbf{q}}+\frac{1}{2}+\frac{1}{2}\eta), \tag{6}$$

$$\Sigma_{\sigma\sigma'}^{>}=\sum_{\eta=+1,-1}\frac{1}{h^3}\int d\mathbf{q}G_{\sigma\sigma'}^{>}(\mathbf{p}+\mathbf{q},E+\eta\Omega_{\mathbf{q}},r)M_{\mathbf{q}}^{2}(N_{\mathbf{q}}+\frac{1}{2}-\frac{1}{2}\eta). \tag{7}$$

The free generalized Kadanoff-Baym (FGKB) [9], [10] ansatz for a two spin band system can be stated as,

$$-iG_{\sigma\sigma'}^{<}(\mathbf{p},E,\mathbf{r},T)=hf_{\sigma\sigma'}(\mathbf{p},\mathbf{r},T)\delta(E-\frac{E_{\sigma}(\mathbf{p})+E_{\sigma'}(\mathbf{p})}{2}) \tag{8}$$

$$iG_{\sigma\sigma'}^{>}(\mathbf{p},E,\mathbf{r},T)=-h(\delta_{\sigma\sigma'}-f_{\sigma\sigma'}(\mathbf{p},\mathbf{r},T))\delta(E-\frac{E_{\sigma}(\mathbf{p})+E_{\sigma'}(\mathbf{p})}{2}) \tag{9}$$

The spin subband Wigner function is found by taking the energy integral of the $G_{\sigma\sigma'}^{<}$:

$$f_{\sigma\sigma'}(\mathbf{p},\mathbf{R},T)=\int dE(-i)G_{\sigma\sigma'}^{<}(\mathbf{p},E;\mathbf{R},T). \tag{10}$$

Substituting the equations (6), (7) to equation (2) together with the FGKB ansatz (8), (9) and using the expression (10) gives the desired set of equations. As a result, the spin up

Wigner function for a Rashba effect RTD in weakly contact with a phonon heat bath can be written as

$$
\begin{aligned}
\hbar\frac{\partial f_{\uparrow\uparrow}(p_{\|},p_z,z,t)}{\partial T} &= \frac{1}{2h}\int dz_1dp_{z2}sin[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})p_z^2f_{\uparrow\uparrow}(p_{\|},p_{z2},z,T)\\
&-\frac{1}{8\pi}\int dz_1dp_{z2}cos[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}+\frac{1}{m^*(z+\frac{z_1}{2})})p_z\frac{\partial}{\partial z}f_{\uparrow\uparrow}(p_{\|},p_{z2},z,T)\\
&-\frac{1}{8\pi}\int dz_1dp_{z2}cos[\frac{1}{\hbar}z_1(p_z-p_{z2})]\frac{\partial}{\partial z}(\frac{1}{m^*(z-\frac{z_1}{2})}+\frac{1}{m^*(z+\frac{z_1}{2})})p_zf_{\uparrow\uparrow}(p_{\|},p_{z2},z,T)\\
&-\frac{1}{8h}\int dz_1dp_{z2}sin[\frac{1}{\hbar}z_1(p_z-p_{z2})]\frac{\partial}{\partial z}(\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})\frac{\partial}{\partial z}f_{\uparrow\uparrow}(p_{\|},p_{z2},z,T)\\
&+\frac{1}{2h}\int dz_1dp_{z2}sin[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})p_{\|}^2f_{\uparrow\uparrow}(p_{\|},p_{z2},z,T)\\
&+\frac{1}{h}\int dz_1dp_{z2}sin(\frac{1}{\hbar}z_1(p_z-p_{z2}))[E_c(z-\frac{z_1}{2})-E_c(z+\frac{z_1}{2})]f_{\uparrow\uparrow}(p_{\|},p_{z2},z,T)\\
&+\frac{\alpha}{\hbar}p_x(f_{\uparrow\downarrow}+f_{\downarrow\uparrow})+i\frac{\alpha}{\hbar}p_y(f_{\uparrow\downarrow}-f_{\downarrow\uparrow})\\
&+\sum_{\sigma=\uparrow,\downarrow}\sum_{\eta=+1,-1}\frac{1}{h^3}\int d\mathbf{q}(\delta_{\uparrow\sigma}-f_{\uparrow\sigma}(\mathbf{p}+\mathbf{q},\mathbf{r},T))f_{\sigma\uparrow}(\mathbf{p},\mathbf{r},T)\\
&\times\delta(\frac{E_\uparrow(\mathbf{p}+\mathbf{q})+E_\sigma(\mathbf{p}+\mathbf{q})}{2}-\frac{E_\sigma(\mathbf{p})+E_\uparrow(\mathbf{p})}{2}+\eta\Omega_{\mathbf{q}})M_{\mathbf{q}}^2(N_{\mathbf{q}}+\frac{1}{2}-\frac{1}{2}\eta)\\
&-\sum_{\sigma=\uparrow,\downarrow}\sum_{\eta=+1,-1}\frac{1}{h^3}\int d\mathbf{q}f_{\uparrow\sigma}(\mathbf{p}+\mathbf{q},\mathbf{r},T)(\delta_{\sigma\uparrow}-f_{\sigma\uparrow}(\mathbf{p},\mathbf{r},T))\\
&\times\delta(\frac{E_\uparrow(\mathbf{p}+\mathbf{q})+E_\sigma(\mathbf{p}+\mathbf{q})}{2}-\frac{E_\sigma(\mathbf{p})+E_\uparrow(\mathbf{p})}{2}+\eta\Omega_{\mathbf{q}})M_{\mathbf{q}}^2(N_{\mathbf{q}}+\frac{1}{2}+\frac{1}{2}\eta). \qquad (11)
\end{aligned}
$$

The first five terms at right hand side of the above equation correspond to the drift terms. These terms are quite complicated comparing to the drift term in the usual single band Wigner function equation since the effective mass is taken to be position dependent. As a result, the effective mass becomes nonlocal. The sixth term gives the potential term and it is calculated self-consistently by coupling to Poisson equation. The seventh term is the subband mixing term. The last two terms correspond to the electron-phonon scattering.

The relaxation time approximation can be made for these. Similarly, the inter spin band polarization of the spin coherence becomes

$$
\begin{aligned}
\hbar\frac{\partial f_{\uparrow\downarrow}(p_\|,p_z,z,t)}{\partial T} &= \frac{1}{2h}\int dz_1dp_{z2}sin[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})p_z^2f_{\uparrow\downarrow}(p_\|,p_{z2},z,T)\\
&-\frac{1}{8\pi}\int dz_1dp_{z2}cos[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}+\frac{1}{m^*(z+\frac{z_1}{2})})p_z\frac{\partial}{\partial z}f_{\uparrow\downarrow}(p_\|,p_{z2},z,T)\\
&-\frac{1}{8\pi}\int dz_1dp_{z2}cos[\frac{1}{\hbar}z_1(p_z-p_{z2})]\frac{\partial}{\partial z}(\frac{1}{m^*(z-\frac{z_1}{2})}+\frac{1}{m^*(z+\frac{z_1}{2})})p_zf_{\uparrow\downarrow}(p_\|,p_{z2},z,T)\\
&-\frac{1}{8h}\int dz_1dp_{z2}sin[\frac{1}{\hbar}z_1(p_z-p_{z2})]\frac{\partial}{\partial z}(\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})\frac{\partial}{\partial z}f_{\uparrow\downarrow}(p_\|,p_{z2},z,T)\\
&+\frac{1}{2h}\int dz_1dp_{z2}sin[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})p_\|^2f_{\uparrow\downarrow}(p_\|,p_{z2},z,T)\\
&+\frac{1}{h}\int dz_1dp_{z2}sin(\frac{1}{\hbar}z_1(p_z-p_{z2}))[E_c(z-\frac{z_1}{2})-E_c(z+\frac{z_1}{2})]f_{\uparrow\downarrow}(p_\|,p_{z2},z,T) \qquad (12)\\
&+\frac{\alpha}{\hbar}p_x(f_{\downarrow\downarrow}-f_{\uparrow\uparrow})+i\frac{\alpha}{\hbar}p_y(f_{\uparrow\uparrow}-f_{\downarrow\downarrow})\\
&+\sum_{\sigma=\uparrow,\downarrow}\sum_{\eta=+1,-1}\frac{1}{h^3}\int d\mathbf{q}(\delta_{\uparrow\sigma}-f_{\uparrow\sigma}(\mathbf{p}+\mathbf{q},\mathbf{r},T))f_{\sigma\downarrow}(\mathbf{p},\mathbf{r},T)\\
&\times\delta(\frac{E_\uparrow(\mathbf{p}+\mathbf{q})+E_\sigma(\mathbf{p}+\mathbf{q})}{2}-\frac{E_\sigma(\mathbf{p})+E_\downarrow(\mathbf{p})}{2}+\eta\Omega_\mathbf{q})M_\mathbf{q}^2(N_\mathbf{q}+\frac{1}{2}-\frac{1}{2}\eta)\\
&-\sum_{\sigma=\uparrow,\downarrow}\sum_{\eta=+1,-1}\frac{1}{h^3}\int d\mathbf{q}f_{\uparrow\sigma}(\mathbf{p}+\mathbf{q},\mathbf{r},T)(\delta_{\sigma\downarrow}-f_{\sigma\downarrow}(\mathbf{p},\mathbf{r},T))\\
&\times\delta(\frac{E_\uparrow(\mathbf{p}+\mathbf{q})+E_\sigma(\mathbf{p}+\mathbf{q})}{2}-\frac{E_\sigma(\mathbf{p})+E_\downarrow(\mathbf{p})}{2}+\eta\Omega_\mathbf{q})M_\mathbf{q}^2(N_\mathbf{q}+\frac{1}{2}+\frac{1}{2}\eta). \qquad (13)
\end{aligned}
$$

Note that $f_{\downarrow\uparrow}=f_{\uparrow\downarrow}^*$. Finally the spin down component of Wigner function is given as,

$$
\begin{aligned}
\hbar\frac{\partial f_{\downarrow\downarrow}(p_\|,p_z,z,t)}{\partial T} &= \frac{1}{2h}\int dz_1dp_{z2}sin[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})p_z^2f_{\downarrow\downarrow}(p_\|,p_{z2},z,T)\\
&-\frac{1}{8\pi}\int dz_1dp_{z2}cos[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}+\frac{1}{m^*(z+\frac{z_1}{2})})p_z\frac{\partial}{\partial z}f_{\downarrow\downarrow}(p_\|,p_{z2},z,T)\\
&-\frac{1}{8\pi}\int dz_1dp_{z2}cos[\frac{1}{\hbar}z_1(p_z-p_{z2})]\frac{\partial}{\partial z}(\frac{1}{m^*(z-\frac{z_1}{2})}+\frac{1}{m^*(z+\frac{z_1}{2})})p_zf_{\downarrow\downarrow}(p_\|,p_{z2},z,T)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{8h}\int dz_1 dp_{z2} sin[\frac{1}{\hbar}z_1(p_z-p_{z2})]\frac{\partial}{\partial z}(\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})\frac{\partial}{\partial z}f_{\downarrow\downarrow}(p_\parallel,p_{z2},z,T)\\
&+\frac{1}{2h}\int dz_1 dp_{z2} sin[\frac{1}{\hbar}z_1(p_z-p_{z2})](\frac{1}{m^*(z-\frac{z_1}{2})}-\frac{1}{m^*(z+\frac{z_1}{2})})p_\parallel^2 f_{\downarrow\downarrow}(p_\parallel,p_{z2},z,T)\\
&+\frac{1}{h}\int dz_1 dp_{z2} sin(\frac{1}{\hbar}z_1(p_z-p_{z2}))[E_c(z-\frac{z_1}{2})-E_c(z+\frac{z_1}{2})]f_{\downarrow\downarrow}(p_\parallel,p_{z2},z,T)\\
&-\frac{\alpha}{\hbar}p_x(f_{\uparrow\downarrow}+f_{\downarrow\uparrow})-i\frac{\alpha}{\hbar}p_y(f_{\uparrow\downarrow}-f_{\downarrow\uparrow})\\
&+\sum_{\sigma=\uparrow,\downarrow}\sum_{\eta=+1,-1}\frac{1}{h^3}\int d\mathbf{q}(\delta_{\downarrow\sigma}-f_{\downarrow\sigma}(\mathbf{p}+\mathbf{q},\mathbf{r},T))f_{\sigma\downarrow}(\mathbf{p},\mathbf{r},T)\\
&\times\delta(\frac{E_\downarrow(\mathbf{p}+\mathbf{q})+E_\sigma(\mathbf{p}+\mathbf{q})}{2}-\frac{E_\sigma(\mathbf{p})+E_\downarrow(\mathbf{p})}{2}+\eta\Omega_{\mathbf{q}})M_{\mathbf{q}}^2(N_{\mathbf{q}}+\frac{1}{2}-\frac{1}{2}\eta)\\
&-\sum_{\sigma=\uparrow,\downarrow}\sum_{\eta=+1,-1}\frac{1}{h^3}\int d\mathbf{q}f_{\downarrow\sigma}(\mathbf{p}+\mathbf{q},\mathbf{r},T)(\delta_{\sigma\downarrow}-f_{\sigma\downarrow}(\mathbf{p},\mathbf{r},T))\\
&\times\delta(\frac{E_\downarrow(\mathbf{p}+\mathbf{q})+E_\sigma(\mathbf{p}+\mathbf{q})}{2}-\frac{E_\sigma(\mathbf{p})+E_\downarrow(\mathbf{p})}{2}+\eta\Omega_{\mathbf{q}})M_{\mathbf{q}}^2(N_{\mathbf{q}}+\frac{1}{2}+\frac{1}{2}\eta). \qquad (14)
\end{aligned}
$$

The equations above can be further simplified by considering a constant effective mass and ignoring the scattering

$$
\begin{aligned}
\hbar\frac{\partial f_{\uparrow\uparrow}(p_\parallel,p_z,z,t)}{\partial T} &= -\frac{p_z}{m^*}\frac{\partial}{\partial z}f_{\uparrow\uparrow}+2\frac{\alpha}{\hbar}p_x\mathrm{Re}[f_{\uparrow\downarrow}]-2\frac{\alpha}{\hbar}p_y\mathrm{Im}[f_{\uparrow\downarrow}]\\
&+\frac{1}{h}\int dz_1 dp_{z2} sin(\frac{1}{\hbar}z_1(p_z-p_{z2}))[E_c(z-\frac{z_1}{2})-E_c(z+\frac{z_1}{2})]f_{\uparrow\uparrow}(p_\parallel,p_{z2},z,T)\\
\hbar\frac{\partial \mathrm{Re}[f_{\uparrow\downarrow}(p_\parallel,p_z,z,t)]}{\partial T} &= -\frac{p_z}{m^*}\frac{\partial}{\partial z}f_{\uparrow\downarrow}+\frac{\alpha}{\hbar}p_x(f_{\downarrow\downarrow}-f_{\uparrow\uparrow})\\
&+\frac{1}{h}\int dz_1 dp_{z2} sin(\frac{1}{\hbar}z_1(p_z-p_{z2}))[E_c(z-\frac{z_1}{2})-E_c(z+\frac{z_1}{2})]f_{\uparrow\downarrow}(p_\parallel,p_{z2},z,T)\\
\hbar\frac{\partial \mathrm{Im}[f_{\uparrow\downarrow}(p_\parallel,p_z,z,t)]}{\partial T} &= -\frac{p_z}{m^*}\frac{\partial}{\partial z}f_{\downarrow\uparrow}+\frac{\alpha}{\hbar}p_y(f_{\uparrow\uparrow}-f_{\downarrow\downarrow})\\
&+\frac{1}{h}\int dz_1 dp_{z2} sin(\frac{1}{\hbar}z_1(p_z-p_{z2}))[E_c(z-\frac{z_1}{2})-E_c(z+\frac{z_1}{2})]f_{\downarrow\uparrow}(p_\parallel,p_{z2},z,T)\\
\hbar\frac{\partial f_{\downarrow\downarrow}(p_\parallel,p_z,z,t)}{\partial T} &= -\frac{p_z}{m^*}\frac{\partial}{\partial z}f_{\downarrow\downarrow}-2\frac{\alpha}{\hbar}p_x\mathrm{Re}[f_{\uparrow\downarrow}]+2\frac{\alpha}{\hbar}p_y\mathrm{Im}[f_{\uparrow\downarrow}]\\
&+\frac{1}{h}\int dz_1 dp_{z2} sin(\frac{1}{\hbar}z_1(p_z-p_{z2}))[E_c(z-\frac{z_1}{2})-E_c(z+\frac{z_1}{2})]f_{\downarrow\downarrow}(p_\parallel,p_{z2},z,T)
\end{aligned}
$$

where $\mathrm{Re}[f_{\uparrow\downarrow}]$ denotes the real part of $f_{\uparrow\downarrow}$ and $\mathrm{Im}[f_{\uparrow\downarrow}]$ denotes the imaginary part of $f_{\uparrow\downarrow}$.

The particle density in each band is written in terms of the diagonal components of the Wigner function matrix as

$$n_\sigma = \frac{1}{h^3}\int d\mathbf{p} f_{\sigma\sigma}(\mathbf{p}, r), \tag{15}$$

so that the spin up and spin down particle density are written as

$$n_\uparrow = \frac{1}{h^3}\int dp_\parallel dp_z f_{\uparrow\uparrow}(p_\parallel, p_z, z), \tag{16}$$

$$n_\downarrow = \frac{1}{h^3}\int dp_\parallel dp_z f_{\downarrow\downarrow}(p_\parallel, p_z, z). \tag{17}$$

The current density can be calculated using the following equation

$$J(r) = \sum_{\sigma,\sigma'}\frac{q}{h^3}\int d\mathbf{p}\frac{\partial H_{\sigma\sigma'}}{\partial \mathbf{p}} f_{\sigma'\sigma}(\mathbf{p}, r). \tag{18}$$

Therefore the spin up and spin down current components can be written respectively as

$$J_\uparrow(z) = \frac{q}{h^3}\int dp_\parallel dp_z \frac{p_z}{m^*} f_{\uparrow\uparrow}(p_\parallel, p_z, z), \tag{19}$$

$$J_\downarrow(z) = \frac{q}{h^3}\int dp_\parallel dp_z \frac{p_z}{m^*} f_{\downarrow\downarrow}(p_\parallel, p_z, z). \tag{20}$$

We derived the two-spin band Wigner function equations for a Rashba effect resonant tunneling diode. The effective mass became nonlocal. This provides the possibility of an accurate study of the relationship between the position dependent effective mass and the

spin splitting in the resonant tunneling structures since the interface effects on the effective mass are explicitly included in this model. Then we simplified the equations using a constant effective mass. This gives a set of equations that are simpler to solve numerically than the first set. The current and particle densities were derived. These equations are going to serve as the starting point for the Wigner function simulations of zero magnetic field resonant spin tunneling devices.

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